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**Abelian subgroup structure of square complex groups and arithmetic of quaternions.**

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The paper under review analyzes the abelian subgroup structure of fundamental groups of certain square complexes of non-positive curvature. Generalizing the method of S. Mozes [Israel J. Math. **90** (1995), no. 1-3, 253–294; MR1336326 (97b:22022)], the authors construct compact lattices  $\Gamma_{p,l}$  of  $G = \mathrm{PGL}_2(\mathbb{Q}_p) \times \mathrm{PGL}_2(\mathbb{Q}_l)$ , where  $p, l$  are distinct primes. A lattice  $\Gamma_{p,l}$  is a finitely presented torsion-free group which acts freely and transitively on the vertices of the affine building  $\Delta$  of  $G$ , with a finite square complex  $\Delta/\Gamma$  as a quotient. The authors show that

- $\Gamma_{p,l}$  is commutative transitive, i.e., the relation of commutativity is transitive on nontrivial elements of  $\Gamma_{p,l}$ ;
- all maximal abelian subgroups  $\Gamma_0$  satisfy  $g\Gamma_0g^{-1} \cap \Gamma_0 = \{1\}$  for all  $g \in \Gamma_{p,l} - \Gamma_0$ .

The authors then further analyze the structure of maximal abelian subgroups and give several explicit examples. *Lucy Lifschitz* (1-OK)

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