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Three amalgams with remarkable normal subgroup structures. (English summary)
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The author constructs three groups, each of which can be decomposed as an amalgamated product $F_9 \star_{F_{81}} F_9$, where F_k denotes the free group of rank k. The first group is simple, the second group is not simple but has no nontrivial normal subgroups of infinite index, and the third group is not simple but has no proper subgroups of finite index. The construction relies on a deep theorem of M. Burger and S. Mozes [Inst. Hautes Études Sci. Publ. Math. No. 92 (2000), 151–194 (2001); MR1839489 (2002i:20042)] which states that certain cocompact lattices in the product of automorphism groups of two locally finite trees cannot have nontrivial normal subgroups of infinite index. This theorem is applied to a cocompact lattice containing a non-residually finite group constructed by D. T. Wise in his Ph.D. Thesis. The explicit presentations of the groups which are obtained are more manageable in practice than in previous constructions of this kind.

Reviewed by *David M. Bundy*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.