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A finitely presented torsion-free simple group. (English summary)
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This paper constructs a finitely presented simple group Γ and gives explicitly its presentation. The proof follows largely the ideas of M. Burger and S. Mozes [C. R. Acad. Sci. Paris Sér. I Math. **324** (1997), no. 7, 747–752; MR1446574 (98g:20041)], but gives an example with manageably small presentation.

D. T. Wise constructed in his Ph.D. thesis ["Non-positively curved squared complexes: aperiodic tilings and non-residually finite groups", Princeton Univ. , Princeton, NJ, 1996] a 7-generated, 12-related group Δ that is not residually finite; more precisely, there exists a nontrivial $w \in \Delta$ that belongs to all finite-index subgroups of Δ.

Rattaggi embeds Δ in a 10-generated, 24-related group Σ, and computes the normal closure of $\langle w \rangle$ in Σ. It has index 4, and is the desired finitely presented simple group Γ.

It is clear that Γ has no finite-index subgroup. On the other hand, a Margulis-type "normal subgroup theorem" implies that Γ has no infinite-index subgroup.

A few bonuses come from the construction: Γ is torsion-free, acts on a product of two trees (it is the fundamental group of a "VH-complex"), has an amalgam decomposition of the form $F_7 *_{F_{73}} F_7$, and is quasi-isometric to a product of two trees.

Reviewed by *Laurent Bartholdi*

References

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.